Real Exchange Rates and Fundamentals in a new Markov-STAR Model *

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Abstract

We propose a new nonlinear Markov-STAR model to capture both the Markov switching and smooth transition dynamics for real exchange rates. We derive stationarity conditions for the model and apply it to the real exchange rates of 17 countries. We relate switching equilibrium rates and volatilities to a set of relevant macroeconomic variables and find, consistent with economic intuitions, that an economy deteriorating relative to the US economy tends to see a significantly increased likelihood of real exchange rate depreciation. Moreover, we document significant connections between rising economic uncertainties and real exchange rate changes as well as exchange rate volatility.

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I. Introduction

In this study we model the time-varying equilibrium as well as the nonlinear adjustment process to the equilibrium of real exchange rates, and link the changing exchange rate equilibrium to economic fundamentals. This approach therefore combines two important features from two major streams of literature modeling real exchange rates: one that focuses on the time-varying equilibrium real exchange rate, albeit in a linear model, and the other that stresses nonlinear mean-reversion but with a constant equilibrium real exchange rate.

Specifically, we introduce a new nonlinear Markov smooth transition autoregressive (Markov-STAR) model to capture both Markov switching (MS) and smooth transition dynamics in real exchange rates. The MS part is modeled using a discrete latent factor and captures the time variations of equilibrium exchange rates and their volatilities, whereas the smooth transition part models nonlinear adjustment of the real exchange rate to the switching equilibrium rate. The discrete MS factor is particularly useful to model sudden but persistent regime changes present in the real exchange rate data. We derive stationarity conditions for the model, outline the estimation algorithm, and apply it to the real exchange rates of 17 countries. We show that this model provides a useful complement to a model that features either MS or smooth transition dynamics alone.

The switching equilibrium rates should reflect the underlying changing economic fundamentals. Therefore, we relate the switching equilibrium rates and volatilities to relevant macroeconomic variables, including the output gap, inflation rate, and economic uncertainty. We document a number of findings that are broadly consistent with standard economic models. First, an economy deteriorating relative to the US economy tends to see a significantly increased likelihood of real exchange rate depreciation relative to the US dollar for most countries in our study. Second, higher economic uncertainty in the United States increases the likelihood of real exchange rate appreciation significantly in many advanced European economies, whereas exactly the opposite is true for some developing countries. Finally, rising economic uncertainty tends to be associated with higher exchange rate volatility across the board.

As such, this study's purpose is twofold. The first is to propose a new nonlinear Markov-STAR model to better capture both types of nonlinear dynamics in real exchange rate data. The other is to further study the connection between the real exchange rate and economic fundamentals through the lens of this new model. The latter makes another important contribution to the literature that has attempted to relate exchange rates to a set of economic fundamental variables in standard economic models (see Engel and West, 2005). Although some previous work has presented supportive empirical evidence in this regard (see Rapach and Wohar, 2002; Cerra and Saxena, 2010), such a close connection is not conclusive (see Bacchetta and van Wincoop, 2013). To the best of our knowledge, our work is the first to study the connection between the equilibrium real exchange rate and economic fundamentals in a highly nonlinear model. Our empirical findings suggest a stronger connection than has been documented in the literature based primarily on linear models.

The remainder of the paper is organized as follows. Section II reviews the literature and compares it with our work. Section III presents an economic motivation. Section IV describes the proposed model. Section V discusses some specification issues for the model, and section VI outlines the estimation of the model parameters. Section VII applies the model to real exchange rate data, and section VIII concludes.

II. Literature review

This section provides a brief review of the literature that has studied nonlinear exchange rate dynamics and the relationship between exchange rates and economic fundamentals. Intuitively, the tendency for the real exchange rate to revert to its equilibrium value becomes stronger only when it is further away from the equilibrium value, because of the transaction and transportation costs involved in arbitrage activities. Consequently, the real exchange rate dynamics in the existing literature often feature such nonlinear adjustments (Taylor and Sarno, 2003; Lo and Zivot, 2001). Michael et al. (1997) and Taylor et al. (2001) apply exponential smooth transition autoregressive (ESTAR) models, proposed by Teräsvirta (1994), to the real exchange rate, and document a stronger tendency for the real exchange rate to revert to its purchasing power parity (PPP) value once this nonlinearity is accounted for. An overview of the econometric properties of this model can be found in van Dijk et al. (2002). Lo (2008) and Lo and Morley (2015) show that certain nonlinear smooth transition models can significantly reduce the half-lives of the PPP deviations and thus have the potential to resolve the PPP puzzle (Rogoff, 1996). Furthermore, Kilian and Taylor (2003) and Rapach and Wohar (2006) show that these smooth transition models can improve forecasts of real exchange rates relative to linear models.

The literature that employs the STAR type of model for real exchange rates often assumes a constant or a stable band for real exchange rates to converge to. There are a number of reasons why this is a very restrictive assumption. For example, Engel (2000) argues that the Balassa-Samuelson effect results in a time-varying equilibrium real exchange rates derived from the productivity differentials of tradable and nontradable sectors. In an effort to capture this effect, Engel and Kim (1999) propose an unobserved components model where they decompose real exchanges rates into a permanent and a transitory component. The permanent component proxies for the time-varying equilibrium real exchange rate and is assumed to follow a random walk. Other studies attempt to account for time variations of equilibrium real exchange rates using different approaches. For example, Hegwood and Papell (1998) allow for multiple exogenous structural breaks in the mean when testing PPP and provide evidence of shorter PPP deviation half-lives. Furthermore, Papell and Prodan (2006) include a time trend together with structural breaks to account for the Balassa-Samuelson effect, and they find stronger evidence for PPP.

We propose an alternative model to capture time-varying equilibrium real exchange rates using a Hamilton (1989) type Markov regime switching process. In this model, a set of latent factors that follow a binomial distribution generates distinct regimes, which turn out to be useful to capture sudden but persistent regime shifts in the real exchange rate data. In particular, Engel and Hamilton (1990) show that an autoregressive model augmented with Markov regime switching means and volatilities successfully captures long swings in real exchange rates. Furthermore, Engel (1994) and Bergman and Hansson (2005) document some evidence that the MS model can improve forecasts of the real exchange rate relative to standard benchmark models.

Different from studies such as Hegwood and Papell (1998), we model shifting regimes in the mean using a MS process to proxy for a time-varying equilibrium real exchange rate. Furthermore, our model also allows for shifts in the variance. Our approach is different from Engel and Kim (1999) in that the equilibrium real exchange rate in our model switches between two regimes persistently, whereas it is constantly moving in theirs. However, the most important difference between our model and those in Engel and Kim (1999) and Hegwood and Papell (1998) is that the transition towards the equilibrium rate is nonlinear and depends on the distance between the actual real exchange rate and the switching equilibrium values. In this way, we combine two important features from two major streams of literature, and we show that both features are important to account for real exchange rate dynamics.[1]

Several recent works attempt to compare and/or combine MS and smooth transition to study the real exchange rate. For example, Kaufmann et al. (2014) argue that the ESTAR model aims to capture the dynamic adjustment process that is self-excited by large deviations from PPP, but the Markov regime-switching model allows for sudden but persistent regime shifts. The latter is better at capturing large external shocks such as currency devaluations. They apply a statistical model specification test and show that

[1]Even a cursory glance shows that the resulting paths from combining both features look rather like a real exchange rate process, unlike the ESTAR- or MS-processes alone.

the former tends to hold for countries within the European Union, whereas the latter effect dominates instead for less developed countries. Ahmad and Lo (2014) study how these two models can be distinguished from each other through a set of Monte Carlo simulations.

Given the above discussion, it is arguable that both types of nonlinear dynamics are present in the real exchange rate data, especially during a relatively long time period. The MS dynamics are better at capturing the possibly changing equilibrium real exchange rate. Threshold or smooth transition dynamics can account for the nonlinear reversion to the equilibrium value. To this end, we propose a new model that combines the Markov regime-switching and the ESTAR approaches in a parsimonious setup. Our new MS-ESTAR model contains an ESTAR process where the inner regime is an MS autoregressive (MSAR) process. The MS component in our model therefore allows for exogenous shocks. However, the size of the shocks is restricted by the forces of the ESTAR component making unrealistic break sizes rather unlikely.

Our approach is in line with Sarno and Valente (2006). They propose a nonlinear MS intercept autoregressive heteroskedastic VECM model containing an MS component and also an exponential adjustment component. However, their model specification is different from ours as they work in a cointegration vector autoregression (VAR) setting. Elliott et al. (2018) consider a hidden MS STAR model with a different estimation approach and present an application to stock indices. By contrast, we provide the economic motivation of such a general model, and we attempt to relate the switching equilibrium real exchange rate to economic fundamentals. We present compelling evidence that the Markov regime switches estimated in our general model are strongly related with underlying economic fundamental variables, such as output gap differentials, inflation differentials, and economic uncertainties, in the right direction. Therefore, our work corroborates recent findings in the literature that support some connections between the exchange rate and economic fundamentals (see for example, Rapach and Wohar, 2002; Cerra and Saxena, 2010; and Balke et al., 2013). What distinguishes our study from these cited studies is that our findings are made in a highly nonlinear but also more realistic model that features both MS and ESTAR adjustments.

III. Economic motivation

This section lays out a simple economic model to motivate our general model that features a time-varying equilibrium real exchange rate. Consider the following uncovered interest parity (UIP) condition:

$$E_t s_{t+1} - s_t = i_t - i_t^* + x_t \tag{1}$$

where s_t is the (log) nominal exchange rate, and i_t is the nominal interest rate at time t. E_t denotes the conditional expectation at time t. The exchange rate s_t is the domestic price of foreign currency. Variables with $*$ denote foreign variables. The variable x_t captures deviations from the UIP, as there has been overwhelming evidence against the strong version of the UIP. One economic interpretation for x_t is the time-varying risk premium within the rational expectation framework.

By definition, the (log) real exchange rate q_t is given by:

$$q_t = s_t + p_t^* - p_t \tag{2}$$

where p denotes the (log) price level. Moreover, by definition, the inflation rate π_t is:

$$\pi_{t+1} = p_{t+1} - p_t \tag{3}$$

and the real interest rate r_t is defined by:

$$r_t = i_t - E_t \pi_{t+1} \tag{4}$$

Combining equations (1), (2), (3), and (4), one can easily derive the following stochastic difference equation that determines the real exchange rate:

$$q_t = E_t q_{t+1} + (r_t^* - r_t) - x_t$$

Iterate this equation forward to obtain:

$$q_t = E_t \sum_{j=0}^{\infty} (r_{t+j}^* - r_{t+j}) - E_t \sum_{j=0}^{\infty} x_{t+j} + \lim_{T \to \infty} E_t q_{t+T}$$

This present value formulation of the exchange rate determination has been explored by Engel and West (2006) and Balke et al. (2013), among many others. In this simple model, the economic fundamentals are the real interest rate differential and risk premium. The last term will be nontrivial if a rational bubble is present. In the presence of nominal rigidity, the real interest rates are primarily affected by productivity in the long run and demand shocks in the short run.

The equilibrium real exchange rate may also change because of the risk premium, which is most likely stationary, see Engel et al. (2009). At the same time, a rational bubble may emerge occasionally. These economic intuitions suggest that we can employ a stochastic MS variable to approximate the time-varying equilibrium real exchange rate, attributed to changing economic fundamentals and/or emergence and disappearance of a rational bubble.[2] Suppose traders collect fundamental news and take advantage of this model or similar ones to execute their trades. They will not start to trade, however, unless the actual exchange rate differs from their calculated equilibrium value by a margin large enough to cover transaction costs. Therefore, reversion to the equilibrium real exchange rate will be nonlinear.

IV. The Markov-STAR model

This section describes elements of our proposed MS-ESTAR model. The general ESTAR model is given by two autoregressive regimes, connected by a smooth exponential transition function $\mathcal{G}(\,\cdot\,;\gamma,c):\mathbb{R}\rightarrow[\,0,1\,]$. This function governs the transition between the two regimes in a smooth way. Alternatively, an ESTAR model can also be interpreted as a continuum of regimes which is passed through by the process.

In general, the univariate ESTAR(p) models, $p\geq 1$ and $d\leq p$, are given by

$$\begin{aligned}(q_t-c)&=\left(\sum_{k=1}^{p}\phi_k(q_{t-k}-c)\right)\times(1-\mathcal{G}(q_{t-d};\gamma,c))+\left(\sum_{k=1}^{p}\theta_k(q_{t-k}-c)\right)\times\mathcal{G}(q_{t-d};\gamma,c)+\varepsilon_t\\&=\sum_{k=1}^{p}\phi_k(q_{t-k}-c)+\left(\sum_{k=1}^{p}\psi_k(q_{t-k}-c)\right)\times\mathcal{G}(q_{t-d};\gamma,c)+\varepsilon_t,\qquad t\geq 1\end{aligned}\tag{5}$$

with $\varepsilon_t\overset{iid}{\sim}(0,\sigma^2)$. Throughout the study we set $d=1$. c can be interpreted as the threshold parameter. As we use demeaned data in our application, we specify the threshold parameter to be the mean of the respective real exchange rate: $c=\overline{q}$.

For an ESTAR model, the transition function $\mathcal{G}(\cdot)$ is given by

$$\mathcal{G}(q_{t-d};\gamma,c)=1-\exp\left\{-\gamma(q_{t-d}-c)^2\right\};\qquad\gamma\geq 0$$

This exponential transition function provides a symmetric adjustment towards the equilibrium. For a survey of the broad field of nonlinear time series models in general and STAR models in particular see van Dijk et al. (2002); see also Teräsvirta (1994).

The most frequently used special case of the general ESTAR model in (5) is the ESTAR(1) model

$$(q_t-c)=\phi(q_{t-1}-c)+\psi(q_{t-1}-c)\left\{1-\exp\left(-\gamma\,(q_{t-1}-c)^2\right)\right\}+\varepsilon_t$$

[2] We employ recurring regimes and thus assume a stationary but very persistent process for the equilibrium real exchange rate.

To model real exchange rate behavior, Taylor et al. (2001) and Rapach and Wohar (2006) impose an inner unit root regime, $\phi = 1$. This regime is corrected back by a white noise process for the outer regime, $\psi = -1$, to ensure global stationarity. In general, stationarity is given as long as $|\phi+\psi| < 1$. Estimation of these models either by nonlinear least squares or maximum likelihood techniques is treated by Klimko and Nelson (1978) and Tjøstheim (1986), respectively.

For the MS component, we use the framework based on Engel and Hamilton (1990) and Engel (1994):

$$q_t = \mu_{s_t} + \varphi_{1s_t} q_{t-1} + \ldots + \varphi_{ps_t} q_{t-p} + \varepsilon_t$$

The values of the autoregressive parameters $\varphi_{1s_t}, \ldots, \varphi_{ps_t}$ and the mean μ_{s_t} and thus the regime switching is governed by an unobservable Markov chain

$$\mathcal{P}(s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots, q_{t-1}, q_{t-2}, \ldots) = \mathcal{P}(s_t = j \mid s_{t-1} = i) = p_{ij}$$

The transition probabilities p_{ij} lie in the open unit interval and $\mu_1 \neq \mu_2 \neq \cdots \neq \mu_N$ to ensure a transient Markov chain and clear identification of the N regimes. The $(N \times N)$ transition probability matrix is then given by

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & p_{ij} & \vdots \\ p_{N1} & \cdots & p_{NN} \end{pmatrix}$$

s_t is assumed to be an ergodic homogeneous Markov chain with invariant probability measure $\pi = (\pi_i)$, and it is initiated at $t = 0$ to guarantee the independence of $(s_t)_{t>0}$. Extensions of this basic framework are possible, see for example, Hamilton and Raj (2002) and the studies cited therein.

The MSAR(1)-ESTAR(1) model (henceforward Markov-STAR) considered in this study is a combination of the MSAR(1)-model and the inner regime of an ESTAR(1) process

$$q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma) q_{t-1} + \varepsilon_{s_t} \tag{6}$$

where ε_t is a white noise error term with $E(\varepsilon_{s_t}) = 0$ and $Var(\varepsilon_{s_t}) = \sigma^2_{s_t}$. This means that we allow only the mean and the variance of the process to switch, whereas the autoregressive parameters are held fixed. Allowing the autoregressive parameters to switch as well would make the model difficult to estimate and the results hard to interpret. It should also be mentioned that the transition function $\mathcal{G}(\cdot)$ is centered around the switching mean, so that the adjustment process depends on the current state of the equilibrium, which is one of the desired properties of our model.

Theorem 1 below describes the stationarity conditions of our model.

Theorem 1 Let $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_{s_t}$ denote a stochastic process being given by the MSAR(1)-ESTAR(1) model in (6) with ε_t being a white noise error term with $E(\varepsilon_{s_t}) = 0$ and $Var(\varepsilon_{s_t}) = \sigma^2_{s_t}$. Let further $\mathcal{G}(q_{t-1}; \gamma)$ be an ESTAR transition function given by $\mathcal{G}(q_{t-1}; \gamma) = 1 - \exp\left(-\gamma q^2_{t-1}\right)$ with $\gamma > 0$. Second order stationarity is then given if $\psi \in (-\phi\mathcal{G} - 1; -\phi\mathcal{G} + 1)$.

Proof See the online appendix.

Remark Following Taylor et al. (2001) and Rapach and Wohar (2006), we impose an inner unit root regime by setting $\phi = 1$. Based on Theorem 1, we therefore set $\psi = -1$ throughout the study to ensure stationarity.

Note that, through variation of γ our model nests an MS only model as well as a random walk model with a switching drift term. Concretely, when $\gamma \to \infty$, the transition function $\mathcal{G}(\cdot)$ returns 1 and the second and third terms in (6) drop out. Hence the model becomes an MS only model. By contrast if $\gamma \to 0$ we have $\mathcal{G}(\cdot) \to 0$ and our model becomes a random walk with switching drift.

Our empirical findings indicate that the half-lives of a one standard deviation shock depend on the smooth transition part, and thus on γ, quite heavily. Further, our estimates for γ show that an MS only model does not seem appropriate for a vast majority of the countries. Hence, we note that the contribution of the smooth transition part in our model is not only given by economic theory alone, but also seems to be important from an empirical point of view. Section VII describes those considerations and the empirical results in more detail.

V. Model specification

This section checks for the validity of our model and provides statistical evidence to distinguish it from Engel and Hamilton (1990)'s and Bergman and Hansson (2005)'s competing models. To do so, we reformulate the models in a random coefficient model environment. This allows us to apply Distaso (2008)'s tests and thereby discriminate between the three models.

For this, consider our model given by (6). Engel and Hamilton (1990)'s model is now given when $\phi = 1$ and $\gamma = 0$, whereas Bergman and Hansson (2005)'s model is given when $|\phi| < 1$ and $\gamma = 0$. Our model applies whenever $\gamma \neq 0$. Now setting $\phi = -\psi$, (6) can then be written as

$$\begin{aligned} q_t &= \mu_{s_t} - \psi \exp(-\gamma q_{t-1}^2) \cdot q_{t-1} + \varepsilon_{s_t} \\ &= \mu_{s_t} + \rho_t \cdot q_{t-1} + \varepsilon_{s_t}, \quad \text{where} \quad \rho_t := -\psi(\exp(-\gamma q_{t-1}^2)) \end{aligned}$$

Hence, if the ESTAR part is added to the MS AR(1) model, the model is located in the class of a random coefficient model with $E(\rho_t) = \rho$ and $V(\rho_t) = \omega^2$. Therefore, testing for $H_0 : \omega^2 = 0$ vs. $H_1 : \omega^2 \neq 0$ is synonymous to testing for adding the ESTAR component. Thus Engel and Hamilton (1990)'s model is met when $\rho_t = 1\ \forall t$, that is, $\omega^2 = 0$ (or $\gamma = 0$). Bergman and Hansson (2005)'s model is met when $|\rho_t| < 1\ \forall t$ with $\omega^2 = 0$ (or $\gamma = 0$). Our model is met whenever $\omega^2 \neq 0$ (or $\gamma \neq 0$).[3]

Distaso (2008) now describes testing the following hypotheses in the model $q_t = \rho_t q_{t-1} + \varepsilon_t$ with a modified LM-test:

(i) $H_0 : \omega^2 = 0$ vs. $H_1 : \omega^2 \neq 0$

(ii) $H_0 : \rho = 1$ vs. $H_1 : |\rho| < 1$

Hypothesis (i) tests, for example, Engel and Hamilton (1990)'s and Bergman and Hansson (2005)'s models against a random coefficient model. The test statistic is

$$ALM^{\omega^2} = \frac{\left(\sum_{t=2}^{T} q_{t-1}^2 \left(q_t - \rho q_{t-1}\right)^2 - \sigma^2\right)^2}{2\sigma^2 \sum_{t=2}^{T} q_{t-1}^4 \left(2\left(q_t - \rho q_{t-1}\right)^2 - \sigma^2\right)} \tag{7}$$

When (i) is true and $\rho = 1$, (7) has a nonstandard limiting distribution. When (i) is true and $|\rho| < 1$, (7) has a χ^2 limiting distribution.

Additionally, hypothesis (ii) tests stationarity within the random coefficient framework. The test statistic is

$$ALM^{\rho} = \frac{\sum_{t=2}^{T} \frac{q_{t-1}\left(q_t - q_{t-1}\right)}{\omega^2 q_{t-1}^2 + \sigma^2}}{\sqrt{\sum_{t=2}^{T} \frac{q_{t-1}^2}{\omega^2 q_{t-1}^2 + \sigma^2}}} \tag{8}$$

When (ii) is true and $\omega^2 = 0$, (8) has a nonstandard limiting distribution. When (ii) is true and $\omega \neq 0$, (8) has a $N(0, 1)$ limiting distribution.

[3]Technically it could also be that $\gamma \neq 0$ and $\psi = 0$, but the latter case seems unrealistic in practice for real exchange rates as q_t would then be a switching mean only model.

Note that in Distaso (2008) the random coefficient ρ_t is assumed to be i.i.d (ρ, ω^2). In our case however, the random coefficients are strongly autocorrelated, because $\rho_t := -\psi(\exp(-\gamma q_{t-1}^2))$ and q_{t-1}^2 exhibits strong autocorrelation. To examine the consequences of the autocorrelated ρ, we conduct a simulation study regarding size and power properties for the LM tests. The results are given in Table 1 in the online appendix, indicating that for the ALM^{ω^2} test the autocorrelation in ρ does not weaken the test properties. In fact, the power of the test even increases for highly autocorrelated ρ.

For the ALM^{ρ} test a further simulation study shows that there are heavy size distortions if ρ is autocorrelated. Hence testing (i) and (ii) sequentially bypasses this problem and yields the following situations:

TABLE 1

Model selection based on Distaso (2008)'s tests.

(i)	(ii)	Model
H_0	H_0	(1): Engel and Hamilton (1990)
H_0	H_1	(2): Bergman and Hansson (2005)
H_1		(3): Our model

Our model is selected whenever the first hypothesis (i) is rejected. In case of a non-rejection in the first test, the model is not a random coefficient model, and thus our model is not supported. Depending on the result of the second test, either Engel and Hamilton (1990)'s or Bergman and Hansson (2005)'s model is chosen by our specification procedure.

VI. Model estimation

Given the two non-linear components, our model has a high flexibility, which makes stable estimation of the model parameters a difficult task. This section describes the estimation steps. The Markov-STAR model (6) is estimated by maximizing the likelihood function

$$L(\mu, P, \sigma; \gamma, c) = \sum_{t=1}^{T} \log f_t \quad \text{with} \quad f_t = \xi_{t|t-1} \cdot \eta_t \tag{9}$$

The $(N\times 1)$ vector $\xi_{t|t-1}$ with elements $\xi_{i,t|t-1} := Pr(s_t = i \,|\, q_{t-1}; \theta)$ for $\theta = (\mu_1, \ldots, \mu_N, \sigma_1, \ldots, \sigma_N, \phi, \psi, \gamma, c)$ denotes the conditional probabilities that the t-th observation lies in regime i. The elements of the $(N \times 1)$ vector η_t are given by

$$\eta_{it} = \left(2\pi\sigma_i^2\right)^{-1/2} \exp\left(-0.5\varepsilon_{it}^2/\sigma_i^2\right) \quad \text{with} \quad \varepsilon_{it} = q_t - \mu_i - \phi q_{t-1} - \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} \tag{10}$$

$\xi_{t|t}$ then describes the $(N \times 1)$ vector of transition probabilities, being constructed by the filter

$$\begin{aligned} \xi_{t|t} &= \frac{\xi_{t|t-1} \odot \eta_t}{\mathbf{1}'(\xi_{t|t-1} \odot \eta_t)} \\ \xi_{t+1|t} &= P \cdot \xi_{t|t} \end{aligned}$$

based on starting values set to N^{-1} in $\xi_{1|0}$, where $\odot$ denotes element-wise multiplication. The smoothed transition probabilities are calculated using Kim (1993)'s algorithm, given by

$$\xi_{t|T} = \xi_{t|t} \odot \left(P' \left(\xi_{t+1|T} \odot \xi_{t+1|t}^{-1} \right) \right) \tag{11}$$

The iteration starts with $\xi_{T|T}$ and goes backwards until $t = 1$. γ is selected using a grid search with values $\gamma_k = (0, 0.01, 0.02, \dots, 1)$. Hence, for each one of the $k = (1, \dots, 101)$ values of γ_k the likelihood is maximized, resulting in $L_k = L(\theta_k)$ with $\theta_k = (\mu_k, P_k, \sigma_k; \gamma_k)$. The final model specification with parameter vector θ_{k^*} then satisfies $k^* = \arg\max_k L_k$.

In the second part of the analysis we specify a logit model to explain the behavior of the transition probabilities by economic fundamental variables X such as output gap differentials, inflation differentials, and economic uncertainty. For this purpose we take the first row of the smoothed probabilities $\xi_{t|T}$, that is, we fix $i = 1$ in $\tilde{p}_i := Pr(s_t = i | q_T; \theta)$. Then we recode the first row of $\xi_{t|T}$ into a binary variable Y such that

$$Y := \begin{cases} 0 & \text{for } \tilde{p}_1 \leq 0.5 \\ 1 & \text{for } \tilde{p}_1 > 0.5 \end{cases} \tag{12}$$

Hence, the Markov chain is expected to be in the second state for $Y = 0$.

We then fit the logit model $Y = f(X\beta + u)$ via maximum likelihood, where u denotes an error term. The estimated coefficients $\hat{\beta}$ then describe the marginal effect of X on $\ln(\tilde{p}_1/(1 - \tilde{p}_1))$. Thus if $\hat{\beta} < 0$, $\tilde{p}_1 < 0.5$ and one would expect an increase of the economic variable leading to a switch from regime 1 to regime 2.

VII. Empirical analysis

Estimation results

This section describes the data and variables used in our estimation and discusses the estimation results of our model. We fit the Markov-STAR model (6) to the real exchange rate data of 17 countries.[4] We use monthly data from 1973-01 until 2019-12 for the real exchange rates, yielding a sample size of at most $T = 564$ (without missing data). Hence, for those countries having adopted the Euro in 1999 (Finland, France, Germany, Italy, the Netherlands, Portugal, and Spain), we have data from 1973-01 until 1998-12, that is, $T = 312$ observations. Additionally, we also provide results for a subsample until 2007-12 prior to the global financial crisis to account for potential structural instability.

The real exchange rates are constructed as $q_t = s_t + p_t^* - p_t$, and demeaned, based on nominal exchange rate data taken from the *IMF eLibrary*, where p_t^* describes the US price level. Hence, as q_t is formulated in direct quotation, an increasing value of q_t corresponds to a depreciation of the currency in real terms relative to the US dollar and vice versa.

For country-specific explanatory variables we compute the output gap and inflation differential, all relative to the United States. Inflation and industrial production data are taken from the *OECD iLibrary*. The output gap variable is constructed by following Engel and West (2006) as the residuals from a quadratically detrended output, where output is measured as the log of seasonally adjusted industrial production. The output gap differential is then computed as the US output gap minus the output gap of the respective country.

Inflation is measured as the first differences of the logarithmic CPI. Inflation differentials are computed as US inflation minus the respective country's inflation rate. Finally, for a measure of US uncertainty we adopt the news-based economic policy uncertainty (EPU) index constructed by Baker et al. (2016).[5]

The results for the model selection procedure are displayed in Table 2 (the subsample results prior to 2008 are shown in Table 2 in the online appendix). The first column of Table 2 indicates that the Markov-STAR model is chosen for approximately one third of all countries, as the hypothesis of a constant coefficient is rejected for five countries. For the subsample analysis, the test chooses our model for seven countries, close to 50% of all the countries. Overall our proposed Markov-STAR model receives reasonable empirical

[4]The countries are selected depending on the data availability of both the exchange rate data as well as the country-specific exogenous explanatory variables.

[5]Another natural choice would be the CBOE Volatility Index (VIX). However, it is only available after 1990.

support, although the support is not uniform.

The parameter estimates of the Markov-STAR model (6) are displayed in Table 3 (and subsample results prior to 2008 in Table 3 in the online appendix). As the real exchange rates are demeaned, a positive (negative) value $\hat{\mu}$ corresponds to a real depreciation (appreciation) of the currency relative to the US dollar over time. Hence, for all countries a regime switch from the first to the second regime marks a real appreciation of the currency. The full-sample results indicate that for more than 80% of the countries the probability of remaining in the same regime is slightly higher in the depreciation regime than in the appreciation regime, as indicated by $\hat{p}_1$ and $\hat{p}_2$. The subsample results are overall very similar to the full-sample results in this regard.

The parameter γ^* dictates the transitional path of the exchange rates toward their equilibrium value. The magnitude of γ^* estimates imply that the transition back to the equilibrium takes place rather smoothly for all countries (except for Turkey, which does not seem to experience any transition dynamics at all). Here we observe values of $\gamma^* \in [0.00, 0.75]$ for all countries.

The smooth transition behaviours of all countries are plotted in Figure 1. First, we observe substantial nonlinear adjustments for all industrialized countries in our sample. An increasing value of the transition function implies a stronger mean reversion as explained in section IV. All European industrialized countries experienced increasing mean reversion between the early to mid-80s, which is approximately the period soon after the European Exchange Rate Regime (ERM) was first created in 1979. Interestingly, there is another notable episode of increasing mean reversion for a number of industrialized European countries, including Finland, Italy, Norway, Spain, Sweden, and the UK around the early 1990s, which corresponds to the 1992 ERM debacle in the UK. More recently toward the end of the sample, the UK, Norway, and Sweden, which all retain their own currencies, seem to experience increasing mean reversion again.

Other industrialized countries such as Canada and Japan display different transition dynamics. Specifically, whereas Japan also experienced increasing mean reversion around the mid-80s, its mean reversion is strongest around the mid-90s right after the collapse of its asset price bubbles. Canada, however, experienced its strongest mean reversion between the early to mid-2000s, which seems to correspond well to the rising global commodity prices during that period (see, for example, Ma et al., 2020) and makes sense given a strong association between its currency and commodities (see, for example, Chen and Rogoff, 2003). Interestingly, three developing countries in our sample (Greece, Mexico, and Turkey) display much less smooth transition dynamics, consistent with Kaufmann et al. (2014)'s findings.

To get a better idea about the contribution of the smooth transition part, we additionally calculate the half-lives of the PPP deviations for those countries. For this we apply Koop et al. (1996)'s methodology for generalized impulse-response functions, that is, we conduct the Monte Carlo integration algorithm with a one standard deviation shock to the real exchange rates. The resulting half-lives are also given in Table 3. Because the subsample results (Table 3 in the online appendix) are very similar to the full-sample results, we focus on the full-sample results for our discussions.

The corresponding columns describe the half-lives implied by the estimated γ^* and the ones resulting if a very small (large) γ of 0.01 (1) is imposed. As one can clearly see, the implied half-lives differ substantially over different values of γ. The difference for the UK, for example, adds up to the value of over 13 years. The higher the value of γ, the faster the shock fades, because the smooth transition part drops out of the model. By contrast, the shock fades out very slowly if we have a small γ, as the transition function becomes smoother in that case. Hence, we can say that the contribution of γ seems to be quite decisive concerning the transition behavior and thus the economic implications. There is a consensus in the literature that half-lives for industrialized countries are in the range of 3 to 5 years (Rogoff, 1996). These rather long half-life estimates have been challenging to reconcile with standard economic models featuring price rigidity. Our results indicate that our model yields notably shorter half-lives using the estimated γ^*. For 11 out of the 14 industrialized countries our estimated half-lives are shorter than the lower bound of the consensus range. The stronger the smooth transition behavior a country displays overall, the shorter the half-life that country displays. These findings are consistent with Lo and Morley (2015) who find that smooth transition models help to shorten the estimated half-lives for G7 countries.

Switching equilibrium rates and economic fundamentals

To explain the switching behavior of equilibrium real exchange rates and relate these switches to economic fundamentals, we compute smoothed probabilities from the estimation results given by (11). We specify the binary variable Y as described in (12). Then we fit a logit model by regressing the binary variable onto various relevant economic fundamentals, including the output gap differential, inflation differential, and the economic uncertainty index. The estimation results are displayed in Table 4 (subsample results prior to 2008 are shown in Table 4 in the online appendix).

Based on the variable definitions in section V, a positive β estimate implies that an increasing value of the fundamental variable increases the likelihood of the currency depreciating in real terms relative to the US dollar, whereas a negative β estimate implies that an increasing value of the fundamental variable increases the likelihood for currency appreciation in real terms relative to the US dollar.

We focus on the full-sample results and discuss the results for the output gap differential first. All of the coefficients except for Greece and Italy are consistent with what economic theory or intuitions would predict, and almost 50% of the positive coefficient estimates are statistically significant. Intuitively, a higher output gap differential, corresponding to a better US economy relative to the studied country, leads to a real appreciation of the US dollar. For those estimates being significant at the 10% level, expectations are met in almost 90% of the cases. The subsample analysis prior to 2008 yields similar results: all significant coefficients remain significant in the subsample and are very close to the full-sample estimates. This shows that the full sample results are quite robust to excluding post-crisis periods, at least when it comes to the output gap differential.

These results are broadly consistent with and strengthen the related results of existing studies in the literature which, however, focus on linear models and nominal exchange rates. For example, Rapach and Wohar (2002) apply the cointegration technique to more than a century of annual observations for 14 industrialized countries and find similar positive correlations between output differentials and nominal exchange rates. Cerra and Saxena (2010) employ the same type of technique to a much broader set of countries and conclude similarly, again by focusing on output differentials and nominal exchange rates. Molodtsova and Papell (2009) and Chinn (2008) also document similar positive relationships between output gap differentials and nominal exchange rates in linear models, but they have to flip the sign of the slope coefficient in the UIP condition (1) to derive their model as noted by Rossi (2013). Importantly, our exercises focus on the switching of the equilibrium real exchange rates instead of actual nominal exchange rates, and thus complement and strengthen the evidence of a positive relationship between output gap differentials and equilibrium real exchange rates.

Concerning inflation, our results are somewhat mixed. In about one third of all significant coefficients in the full sample and half of all significant coefficients in the subsample, the sign of $\hat{\beta}$ is negative, implying that an increasing inflation differential increases the likelihood of the currency appreciating in real terms relative to the US dollar. The subsample analysis also yields slightly fewer significant coefficients. It is unclear how inflation differentials may affect equilibrium real exchange rate given that the latter depends upon long-term factors such as productivity. Other studies including Rapach and Wohar (2002) and Cerra and Saxena (2010) focus on monetary aggregates instead of inflation rates by basing their empirical exercises on the classical monetary model of nominal exchange rates.

Finally, economic uncertainty is considered by including the EPU index in the logit regression. This variable is statistically significant in 11 of the 17 countries under investigation, and thus seems important in describing the switching behavior of the real exchange rate. To the best of our knowledge the literature has yet to yield a clear prediction of the directional effect of economic uncertainty on exchange rate changes. Intuitively an increase in economic uncertainty affects all currencies, and thus results in a higher risk premium for all currencies. As such, it is unclear whether a particular currency will appreciate or depreciate against the US dollar. However, if a currency is affected more (less) than the benchmark currency, the US dollar, then the currency is expected to depreciate (appreciate) relative to the US dollar.

The regression results in Table 4 seem to suggest that rising US economic uncertainty appears to affect a number of countries, all industrialized, namely France, Germany, Greece, Japan, the Netherlands, and Portugal, less than the United States, so that it tends to raise the likelihood for these currencies to appreciate in real terms given the negative $\hat{\beta}$. By contrast, for several other countries (namely Brazil, Canada, Sweden, and Turkey), rising US economic uncertainty tends to lead their currencies to depreciate in real terms relative to the US dollar, suggesting that their economies are more affected by US uncertainty.[6] The subsample results are broadly similar to the full-sample results, suggesting that the relationship between economic uncertainty and switches is stable over time.

Although our model allows both the mean and volatility to switch between the two regimes, one drawback is that we have only one latent MS factor to govern both the mean and volatility switches. This restriction is solely for parsimony.[7] Despite this limitation, our model does allow a possible investigation of the relationship between volatility switches and economic fundamentals. This is particularly relevant when relating switches to the uncertainty index. In other words, one can also interpret the connection between exchange rate regime switches and the EPU index as reflecting a close association between general economic uncertainty and exchange rate volatility.

For example, the estimation results for Brazil in Table 3 show that its currency volatility becomes significantly lower switching from regime 1 (high equilibrium rate) to regime 2 (low equilibrium rate). Therefore, the significantly positive coefficient estimate associated with the EPU index for Brazil in Table 4 implies that a rising economic uncertainty index is closely related to an elevating currency risk in exchange rate markets. The same logic applies to other developing country currencies (Mexico and Turkey) that appear to experience significant volatility switches. However, this pattern fails to emerge in general.

To investigate to what extent this is because of the drawback of mixing switches of mean and volatility as mentioned above, we modify (6) such that μ is fixed whereas σ

[6]Note that Canada's economy relies heavily on the production and export of oil, whereas Brazil and Turkey are developing economies. Our finding that the economic uncertainty index has different effects on developed and developing countries is similar to Bansal and Dahlquist (2000), who find that the forward premium puzzle is limited only to developed countries.

[7]We leave relaxing this restriction to allow for more general switches to future research.

is still allowed to switch, to focus on the volatility switching. The estimation results of this model are displayed in Table 5. Table 6 presents the corresponding logit estimation results for the case of a fixed μ. Following the same specification as before, $\hat{\beta} < 0$ implies that an increasing economic fundamental variable increases the likelihood for the currency being in regime 2, whereas $\hat{\beta} > 0$ implies an increasing likelihood of being in regime 1. Based on the results from Table 5, a switch from regime 1 to regime 2 corresponds to an increase in volatility.

If we focus on the results associated with the economic uncertainty index in Table 6, we have statistically significant coefficient estimates in 10 out of the 17 countries under investigation. Furthermore, except for Denmark, in all significant cases the coefficient estimates are negative, and thus suggest that rising economic uncertainty tends to increase the likelihood of switching from the low volatility regime 1 to the high volatility regime 2, remarkably consistent with economic intuitions. The subsample analysis produces even better results with 10 statistically significant and negative coefficient estimates in 11 countries, which mostly match those significant countries in the full sample.

Finally, in our most general model where both mean and volatility switches are controlled by the same latent MS factor, we want to ensure that the switching is not entirely driven by the volatility switching. Therefore, we estimate (6) by fixing σ and present the results in Table 7. We then use a likelihood ratio (LR) test to compare the model with the restricted σ with the general model with the unrestricted σ. Hence, we calculate $LR = -2\ln(L_{k^{**}}/L_{k^{*}})$, where $L_{k^{**}}$ ($L_{k^{*}}$) denotes the maximized value of (9) in the restricted (unrestricted) model. The test statistics are given in the last column of Table 7. All LR values are below 1 and fairly small. With a critical value of 2.71 for $\alpha = 0.1$, the Null of $H_0 : \sigma_2 = \sigma_1$ can never be rejected. Hence, we can conclude that it is rather the switch in mean than the switch in volatility that drives the real exchange rate dynamics in the most general model that we estimate.

Out-of-sample forecasts

Given the aforementioned empirical evidence in support of our Markov-STAR model, it is natural to evaluate its out-of-sample forecast performance relative to a benchmark model such as the naive random walk model as in Meese and Rogoff (1983). Table 8 presents mean squared errors for 1-period-ahead forecasts of the real exchange rate starting at 1990, using our MS-ESTAR model and random walk models. For the Markov-STAR model, we experiment with both fixing the regime for the upcoming period using 0.5 as the cutoff value for the regime probability and averaging predictions of both regimes weighted by each regime's probability. We report results for both the random walks without drift and with drift. Overall, the Markov-STAR model with averaged regime predictions performs slightly better than the Markov-STAR with a fixed regime. However, mean squared errors of the random walk models are evidently smaller than those of our Markov-STAR models, reminiscent of results in Meese and Rogoff (1983). Nevertheless, these results cannot be readily taken as evidence against MS-ESTAR models. First, it is notoriously difficulty for these nonlinear models to beat a naive random walk model over short horizons, when mean reversions derived from economic fundamentals have yet to take place. Second, as Kilian and Taylor (2003) forcefully illustrate, the out-of-sample tests of these nonlinear models typically have a very low power against a naive random walk model, given the short span of available post-Bretton Woods periods.

VIII. Conclusion

Motivated by popular economic models, we propose a new nonlinear Markov-STAR model to capture both the time-varying real exchange rate equilibrium and volatilities – driven by various economic factors – and the smooth transition type of nonlinear adjustment to the equilibrium – due to the economic intuitions that imply arbitrage profits only beyond certain transaction and transportation costs. We find that this Markov-STAR model can better capture the time series dynamics of the real exchange rate and the implied half-lives. More importantly, we aim to evaluate the connection between the real exchange rate and its economic fundamentals through the lens of this newly-proposed nonlinear model.

The connection between the real exchange rate and its economic fundamentals is featured in most economic models, but the empirical evidence to establish such a close relationship has not been conclusive. To the best of our knowledge, our work is the first to investigate such a connection in a highly nonlinear context, which considers more realistic nonlinearities in exchange rate data. We use the United States as the benchmark country and apply our model to 17 countries.

We find strong evidence that the varying equilibrium real exchange rate is closely related to the economic fundamentals predicted by standard economic models. Specifically, we find that an economy deteriorating relative to the US economy tends to increase the likelihood of the real exchange rate depreciating relative to the US dollar significantly. Our exercises also cast light on the role of economic uncertainty in affecting the equilibrium exchange rates. We find that a higher economic uncertainty index in the United States increases the likelihood of a real exchange rate appreciation relative to the US dollar significantly for many advanced European economies, whereas it is the opposite for some developing countries. Last, but not least, we interestingly document compelling evidence that rising economic uncertainty tends to be associated with higher exchange rate volatility, consistent with the usual economic intuitions.

Based on these findings, we reach our conclusion that the connection between the exchange rates and their economic fundamentals becomes much stronger and clearer once the nonlinearities of real exchange rates are explicitly accounted for. Therefore, although our work provides additional empirical support for the fundamental approach to the real exchange rate, it also points out the importance of including realistic nonlinearities into standard economic models of the real exchange rate.

TABLE 2

Results for the modified LM-tests on model specification

		(i)			*(ii)*			
Country	ALM^{ω^2}	$\hat{\rho}$	$\hat{\sigma}_\varepsilon^2$	ALM^{ρ}	$\hat{\sigma}_\varepsilon^2$	$\hat{\omega}^2$	*Model*	*T*
Brazil	**0.57**	0.98865	0.00156	**-1.48**	0.00149	0.00116	1	481
Canada	**0.01**	0.99133	0.00022	**-1.55**	0.00022	0.00027	1	564
Denmark	**1.33**	0.98581	0.00063	-1.88	0.00059	0.00190	2	564
Finland	**1.34**	0.98140	0.00063	-1.59	0.00055	0.00442	2	312
France	1.68	0.98306	0.00072				3	312
Germany	**1.09**	0.98427	0.00080	**-1.52**	0.00075	0.00228	1	312
Greece	**0.01**	0.98572	0.00076	**-1.43**	0.00076	0.00000	1	336
Italy	4.66	0.98342	0.00065				3	312
Japan	**0.49**	0.98768	0.00076	-1.94	0.00073	0.00092	2	564
Mexico	14.93	0.96815	0.00179				3	564
Netherlands	**1.06**	0.98203	0.00078	-1.73	0.00073	0.00214	2	312
Norway	**0.49**	0.98365	0.00059	-1.98	0.00056	0.00194	2	564
Portugal	**0.53**	0.98844	0.00086	**-1.21**	0.00078	0.00234	1	312
Spain	**1.05**	0.98788	0.00067	**-1.33**	0.00061	0.00219	1	312
Sweden	**0.71**	0.98731	0.00064	-1.73	0.00060	0.00156	2	564
Turkey	24.01	0.98817	0.00157				3	564
UK	7.19	0.98038	0.00061				3	564

Notes: States the test statistics (7) and (8) and underlying parameter estimates. Hypothesis (i): Based on critical values of Table 1 in Distaso (2008). Hypothesis (ii): Based on critical values of Table 3 in Distaso (2008). Bold values indicate that the Null cannot be rejected for $\alpha = 0.1$.

TABLE 3

Estimation results and half-lives of the Markov-STAR model

Country	*Parameter estimates* $\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*	*Half-lives* $\gamma = \gamma^*$	$\gamma = 0.01$	$\gamma = 1$	*St. dev.*
Brazil	0.0035	-0.0017	0.8829	0.9836	0.0599	0.0112	0.03	7.3367	12.2233	1.2329	0.2659
Canada	0.0131	-0.0103	0.9235	0.9366	0.0099	0.0092	0.72	2.8740	18.2583	2.5272	0.1221
Denmark	0.0176	-0.0239	0.9437	0.9386	0.0151	0.0138	0.06	4.7767	10.6606	1.4946	0.1524
Finland	0.0204	-0.0203	0.9340	0.9452	0.0169	0.0128	0.64	1.9243	11.4317	1.6299	0.1437
France	0.0193	-0.0249	0.9404	0.9480	0.0176	0.0135	0.21	2.6796	10.3293	1.4219	0.1500
Germany	0.0227	-0.0234	0.9414	0.9295	0.0165	0.0168	0.15	3.1253	10.3554	1.4581	0.1639
Greece	0.0308	-0.0151	0.9360	0.9447	0.0172	0.0162	0.24	2.7640	9.1678	1.5735	0.1524
Italy	0.0273	-0.0149	0.9497	0.9528	0.0164	0.0152	0.43	2.3011	9.9631	1.6638	0.1459
Japan	0.0157	-0.0291	0.9408	0.9168	0.0160	0.0190	0.27	2.3731	8.2479	1.3253	0.1974
Mexico	0.0224	-0.0057	0.8678	0.9866	0.0849	0.0095	0.04	3.4689	5.4364	1.2924	0.1686
Netherlands	0.0202	-0.0264	0.9433	0.9516	0.0171	0.0140	0.14	2.9546	9.8606	1.3608	0.1573
Norway	0.0206	-0.0188	0.9329	0.9389	0.0151	0.0138	0.65	2.0117	12.6194	1.6676	0.1366
Portugal	0.0187	-0.0278	0.9197	0.9381	0.0207	0.0143	0.06	4.8307	10.5453	1.4125	0.1975
Spain	0.0274	-0.0162	0.9322	0.9505	0.0174	0.0146	0.30	2.5079	9.0485	1.5838	0.1846
Sweden	0.0213	-0.0204	0.9435	0.9437	0.0167	0.0134	0.32	2.4449	10.9774	1.5339	0.2085
Turkey	0.0219	-0.0062	0.8597	0.9664	0.0709	0.0170	0.00	41.6667	5.7956	1.1022	0.2525
UK	0.0193	-0.0190	0.9241	0.9282	0.0160	0.0146	0.42	2.3525	13.5592	1.7138	0.1241

Notes: Estimation of model (6): $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_{s_t}$ with $\phi = 1$ and $\psi = -1$. Half-lives in years for a one standard deviation shock, where the first refers to the estimated value of γ, the second with an imposed γ of 0.01, and the third with an imposed γ of 1. Empirical standard deviations are given in the last column.

TABLE 4

Logit regression results

Country	*Constant*	*p-value*	*Outp. gap*	*p-value*	*Inflation*	*p-value*	*Uncert.*	*p-value*	*McF* R^2	*T*
Brazil	**-1.0945**	0.0004	1.5367	0.1082	-0.0199	0.8320	**0.0098**	0.0001	0.0517	469
Canada	**-0.9133**	0.0005	**4.4545**	0.0978	**35.2524**	0.0000	**0.0077**	0.0009	0.0764	552
Denmark	**0.8251**	0.0014	**3.4655**	0.0065	**16.3281**	0.0008	-0.0034	0.1227	0.0469	552
Finland	0.5419	0.2666	1.5027	0.3706	0.3295	0.9319	-0.0051	0.3025	0.0440	300
France	**2.2238**	0.0001	1.0956	0.7481	**-12.0232**	0.0315	**-0.0197**	0.0004	0.0922	300
Germany	**1.2517**	0.0194	**3.6366**	0.0420	-3.6938	0.4866	**-0.0119**	0.0226	0.0675	300
Greece	0.4924	0.3505	**-5.7487**	0.0017	3.2790	0.2606	**-0.0119**	0.0409	0.0678	324
Italy	-0.1834	0.7355	-2.0192	0.2875	-2.7342	0.4853	-0.0068	0.2013	0.0414	300
Japan	**1.3140**	0.0000	**2.8782**	0.0007	4.0520	0.2683	**-0.0066**	0.0032	0.0489	552
Mexico	**-2.6889**	0.0000	0.5987	0.7574	**-0.9353**	0.0493	**0.0134**	0.0000	0.1855	458
Netherlands	**2.2630**	0.0000	**8.1622**	0.0039	-4.7219	0.3907	**-0.0181**	0.0006	0.0991	300
Norway	0.0686	0.7837	0.6924	0.2194	2.2603	0.5324	0.0001	0.9605	0.0240	552
Portugal	**2.0515**	0.0001	0.5564	0.5157	0.9297	0.6490	**-0.0140**	0.0106	0.0678	300
Spain	-0.3191	0.5433	1.7140	0.5227	1.3699	0.6632	-0.0015	0.7671	0.0310	300
Sweden	**-0.6211**	0.0161	**4.0092**	0.0088	**10.3657**	0.0056	**0.0066**	0.0038	0.0500	552
Turkey	**-2.3771**	0.0000	**1.8798**	0.0401	0.6440	0.2942	**0.0111**	0.0000	0.2749	420
UK	-0.1932	0.4567	1.0250	0.5648	**4.9820**	0.0992	0.0023	0.2992	0.0275	552

Notes: Based on (12) for the smoothed probabilities of model (6). Significant coefficients for $\alpha = 0.1$ in bold.

TABLE 5

Estimation results of the Markov-STAR model with μ fixed

Country	$\hat{\mu}$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*
Brazil	-0.0012	0.9836	0.8805	0.0110	0.0601	0.00
Canada	-0.0004	0.9792	0.8932	0.0036	0.0186	0.00
Denmark	0.0024	0.9902	0.9012	0.0072	0.0306	0.13
Finland	-0.0031	0.9908	0.8968	0.0087	0.0329	0.29
France	0.0016	0.9748	0.8776	0.0105	0.0361	0.00
Germany	-0.0005	0.9787	0.8805	0.0126	0.0392	0.00
Greece	-0.0028	0.9846	0.8917	0.0053	0.0339	0.09
Italy	-0.0043	0.9868	0.9114	0.0055	0.0322	0.38
Japan	0.0022	0.9990	0.8964	0.0046	0.0330	0.23
Mexico	-0.0055	0.9859	0.8665	0.0095	0.0893	0.03
Netherlands	-0.0011	0.9724	0.8804	0.0128	0.0381	0.02
Norway	-0.0004	0.9738	0.8850	0.0079	0.0313	0.12
Portugal	0.0013	0.9821	0.8760	0.0099	0.0398	0.07
Spain	-0.0039	0.9714	0.8765	0.0099	0.0366	0.01
Sweden	0.0009	0.9697	0.8747	0.0082	0.0331	0.01
Turkey	-0.0037	0.9668	0.8656	0.0167	0.0738	0.00
UK	0.0012	0.9653	0.8727	0.0084	0.0336	0.76

Notes: Estimation of model (6), where now $q_t = \boldsymbol{\mu_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_{s_t}$ with $\phi = 1$ and $\psi = -1$.

TABLE 6

Logit regression results for μ fixed

Country	*Constant*	*p-value*	*Outp. gap*	*p-value*	*Inflation*	*p-value*	*Uncert.*	*p-value*	*McF* R^2	*T*
Brazil	**1.1120**	0.0003	-1.2745	0.1823	0.0443	0.6377	**-0.0100**	0.0001	0.0516	469
Canada	**-0.8258**	0.0093	-4.1847	0.1784	**-17.0390**	0.0170	-0.0046	0.1088	0.0634	552
Denmark	**-2.1946**	0.0000	2.4090	0.1313	**23.6150**	0.0011	**0.0063**	0.0118	0.0495	552
Finland	-0.0893	0.8744	**6.8182**	0.0003	**-10.4111**	0.0120	-0.0086	0.1380	0.1104	300
France	**1.0874**	0.0540	**7.5973**	0.0402	-2.9856	0.6043	**-0.0183**	0.0023	0.0773	300
Germany	**1.2090**	0.0271	**4.4693**	0.0135	-0.1029	0.9848	**-0.0141**	0.0089	0.0753	300
Greece	**-1.2017**	0.0633	**6.3143**	0.0000	1.9805	0.5581	-0.0028	0.7022	0.1075	324
Italy	0.7243	0.3108	**10.5070**	0.0000	-0.6965	0.8782	**-0.0201**	0.0062	0.1681	300
Japan	**-1.8689**	0.0000	0.8691	0.4979	**-8.3296**	0.0641	-0.0003	0.9401	0.0400	552
Mexico	**2.6674**	0.0000	-0.3291	0.8657	**0.9290**	0.0521	**-0.0130**	0.0000	0.1835	458
Netherlands	**0.9341**	0.0576	-2.1953	0.3929	-5.7840	0.2786	**-0.0110**	0.0293	0.0507	300
Norway	-0.3477	0.2148	**2.3427**	0.0002	2.3207	0.5561	**-0.0049**	0.0521	0.0389	552
Portugal	0.7233	0.1638	0.3226	0.6981	**5.8238**	0.0066	-0.0075	0.1727	0.0582	300
Spain	**1.3360**	0.0181	**5.7841**	0.0361	-2.0700	0.5098	**-0.0182**	0.0015	0.0957	300
Sweden	**-0.5435**	0.0469	**4.1312**	0.0115	2.2769	0.5604	-0.0022	0.3514	0.0263	552
Turkey	**2.4950**	0.0000	**-3.0262**	0.0009	-0.7868	0.1962	**-0.0130**	0.0000	0.3049	420
UK	**-0.5726**	0.0295	1.2092	0.5084	5.2638	0.1032	0.0017	0.4427	0.0226	552

Notes: Based on (12) for the smoothed probabilities of model (6), where μ_t is fixed. Significant coefficients for $\alpha = 0.1$ in bold.

Table 7

Estimation results of the Markov-STAR model with σ fixed

Country	$\hat{\mu}$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}$	γ^*	*LR*
Brazil	0.0888	-0.0073	0.9435	0.9838	0.0291	0.11	0.17
Canada	0.0131	-0.0102	0.9276	0.9333	0.0095	0.62	0.01
Denmark	0.0192	-0.0221	0.9438	0.9319	0.0145	0.13	0.00
Finland	0.0216	-0.0191	0.9388	0.9288	0.0150	0.61	0.04
France	0.0217	-0.0222	0.9401	0.9245	0.0158	0.20	0.02
Germany	0.0220	-0.0240	0.9431	0.9317	0.0167	0.13	0.00
Greece	0.0311	-0.0149	0.9401	0.9443	0.0166	0.25	0.01
Italy	0.0279	-0.0145	0.9529	0.9531	0.0157	0.43	0.03
Japan	0.0145	-0.0311	0.9409	0.9247	0.0173	0.30	0.02
Mexico	0.3597	-0.0014	0.9990	0.9990	0.0292	0.28	0.43
Netherlands	0.0207	-0.0258	0.9504	0.9366	0.0158	0.11	0.01
Norway	0.0211	-0.0184	0.9348	0.9352	0.0144	0.62	0.01
Portugal	0.0327	-0.0153	0.9348	0.9428	0.0191	0.18	0.02
Spain	0.0294	-0.0150	0.9385	0.9529	0.0157	0.29	0.04
Sweden	0.0230	-0.0187	0.9501	0.9387	0.0153	0.34	0.03
Turkey	0.1633	-0.0036	0.9911	0.9967	0.0301	0.03	0.11
UK	0.0197	-0.0186	0.9283	0.9248	0.0154	0.42	0.01

Notes: Estimation of model (6), where now $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \boldsymbol{\varepsilon_t}$ with $\phi = 1$ and $\psi = -1$.

Table 8

Mean squared errors for 1-period-ahead forecasts

Country	*Markov-STAR (fixed regimes)*	*Markov-STAR (averaged)*	*Random walk (without)*	*Random walk (with drift)*
Brazil	1.1863	1.1853	0.6161	0.6142
Canada	0.1301	0.1192	0.1012	0.1008
Denmark	0.3269	0.2632	0.1915	0.1913
Finland	0.1140	0.1022	0.0869	0.0874
France	0.1146	0.0956	0.0636	0.0634
Germany	0.1306	0.0992	0.0667	0.0664
Greece	0.1408	0.1170	0.0839	0.0836
Italy	0.1155	0.1032	0.0781	0.0779
Japan	0.4317	0.3760	0.2438	0.2445
Mexico	0.4418	0.4065	0.3470	0.3460
Netherlands	0.1290	0.1107	0.0718	0.0716
Norway	0.3178	0.2737	0.2153	0.2149
Portugal	0.1273	0.1042	0.0701	0.0696
Spain	0.1306	0.1137	0.0790	0.0793
Sweden	0.4436	0.4128	0.2443	0.2430
Turkey	0.6444	0.6295	0.5322	0.5306
UK	0.3434	0.2582	0.1890	0.1885

Notes: Forecasting period starts in 1990. For the model $\hat{q}_{t+1} = \hat{\mu}_t + \phi q_t + \psi \mathcal{G}(q_t; \hat{\gamma}, c) q_t$ with $\phi = 1$ and $\psi = -1$, where $\hat{\mu}_t$ is based on a fixed regime if $\hat{\xi}_{i,t+1|T} \geq .5$, or a $\hat{\xi}_{t+1|T}$-weighted average. Compared to the naive approach $\hat{q}_{t+1} = q_t$ and a random walk with drift $\hat{q}_{t+1} = \alpha + q_t$.

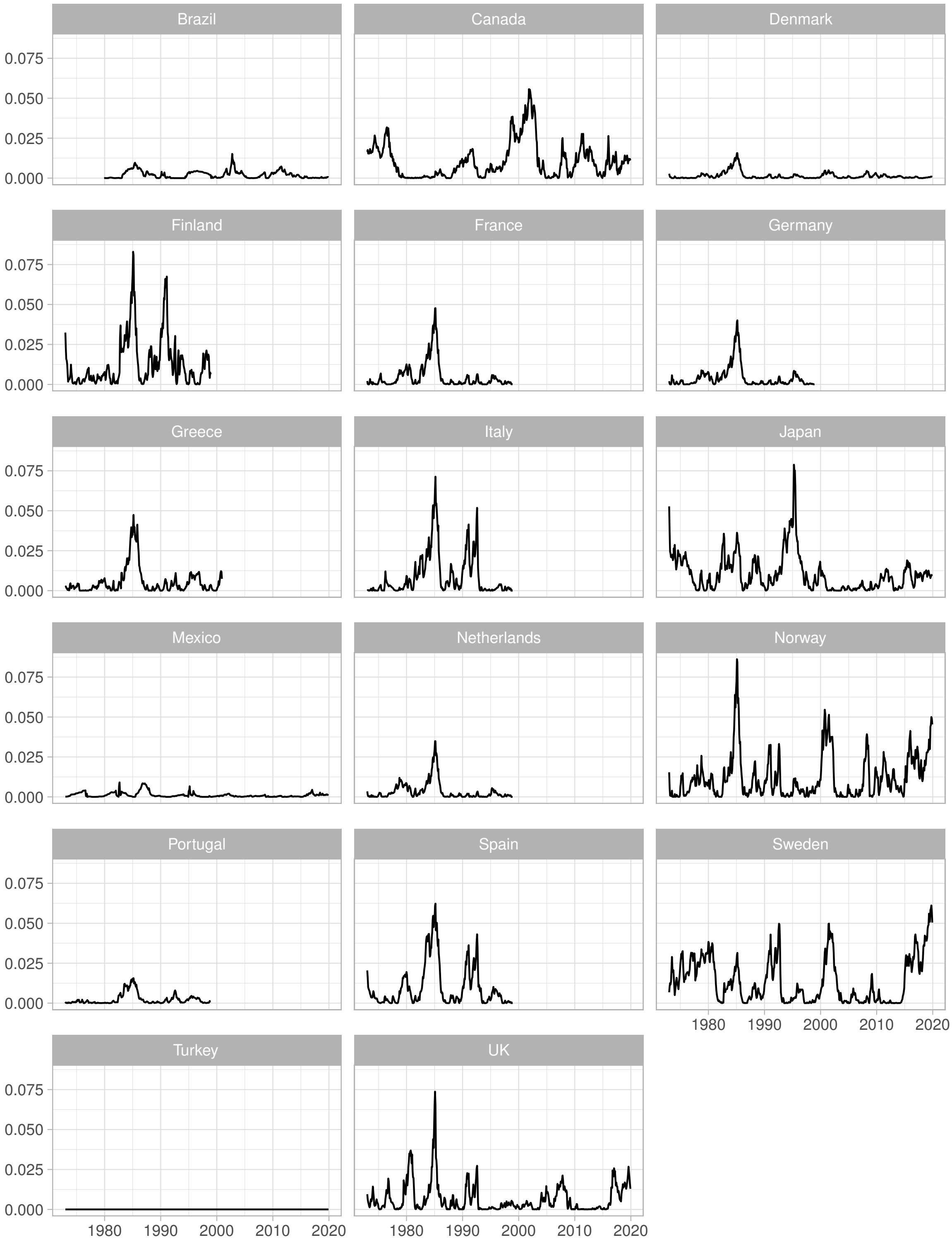

Figure 1. Estimated transition functions $\mathcal{G}(q_{t-1}; \gamma^*, c)$ of model (6)

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