

Online Appendix

Appendix A: Proof of Theorem 1

In this section we give the proof of Theorem 1 in section IV. of the paper.

Proof Let q_t follow an univariate markov switching $AR(p)$ model given by

$$q_t = c(\Delta_t) + \sum_{i=1}^p a_i(\Delta_t)q_{t-i} + \varepsilon_t \quad (1)$$

where (Δ_t) is an irreducible, aperiodic Markov chain with finite state space $\{1, 2, \dots, d\}$ and stationary transition probabilities denoted $p_{ij} = P(\Delta_t = j | \Delta_t = i)$. Further let (Δ_t) be stationary, with stationary probabilities denoted by $\pi_i = P(\Delta_1 = i)$, $1 \leq i \leq d$.

(1) can now be written as

$$z_t = \omega_t + \Phi_t z_{t-1} = c_t + \varepsilon_t + \Phi_t z_{t-1} = c_t + \Sigma_t \eta_t + \Phi_t z_{t-1} \quad (2)$$

where

$$c_t = \begin{pmatrix} c(\Delta_t) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, z_t = \begin{pmatrix} q_t \\ q_{t-1} \\ \vdots \\ q_{t-p+1} \end{pmatrix}, \Sigma_t = \begin{pmatrix} \sigma(\Delta_t) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \Phi_t = \begin{pmatrix} a_1(\Delta_t) & a_2(\Delta_t) & \cdots & a_p(\Delta_t) \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Francq and J.-M.Zakoian (2001) show that the unique stationary solution of (2) is given by

$$z_t = \omega_t + \sum_{k=1}^{\infty} \Phi_t \Phi_{t-1} \cdots \Phi_{t-k+1} \omega_{t-k} \quad (3)$$

whenever the Lyapunov exponent γ defined by

$$\gamma = \inf_t \left\{ E \left[t^{-1} \log \|\Phi_t \Phi_{t-1} \cdots \Phi_1\| \right] \right\}$$

is strictly negative.

To show second order stationarity, it suffices to show that (3) converges to zero in L^2 at an exponential rate, as $k \rightarrow \infty$. As $E[\text{vec}(z_t z_t')] = \mathbf{1}' P^k \cdot (S \cdot \text{vec}(\Omega) + C)$, P^k converges to zero at an exponential rate as k goes to infinity under the condition that $\rho(\tilde{P}) < 1$, where

$\rho(M)$ denotes the spectral radius of any matrix M ($\rho(M) = \max_\lambda |\lambda|$) and

$$\tilde{P} = \begin{pmatrix} p_{11}(\Phi(1) \otimes \Phi(1)) & p_{21}(\Phi(1) \otimes \Phi(1)) & \cdots & p_{d1}(\Phi(1) \otimes \Phi(1)) \\ p_{12}(\Phi(2) \otimes \Phi(2)) & p_{22}(\Phi(2) \otimes \Phi(2)) & \cdots & p_{d2}(\Phi(2) \otimes \Phi(2)) \\ \vdots & \vdots & \vdots & \vdots \\ p_{1d}(\Phi(d) \otimes \Phi(d)) & p_{2d}(\Phi(d) \otimes \Phi(d)) & \cdots & p_{dd}(\Phi(d) \otimes \Phi(d)) \end{pmatrix} \quad (4)$$

where $\Phi(k)$ denotes the matrix obtained by replacing Δ_t by k in Φ_t .

Our model is now given by

$$\begin{aligned} q_t &= c(\Delta_t) + \sum_{i=1}^p \psi_i q_{t-i} + \left(\sum_{i=1}^p \phi_i q_{t-i} \right) \mathcal{G}(q_{t-i}; \gamma) + \varepsilon_t \\ &= c(\Delta_t) + \sum_{i=1}^p (\psi_i + \phi_i \mathcal{G}(q_{t-i}; \gamma)) q_{t-i} + \varepsilon_t \\ &= c(\Delta_t) + \sum_{i=1}^p \alpha_i q_{t-i} + \varepsilon_t \end{aligned}$$

where $\alpha_i = \psi_i + \phi_i \mathcal{G}(q_{t-i}; \gamma)$ and $\mathcal{G}(\cdot; \gamma, c) = 1 - \exp\{-\gamma(q_{t-i})^2\}$; $\gamma > 0$. Hence our model is given by (1) with $a_i(\Delta_t) = \alpha_i \forall t \geq 1$. Thus (4) simplifies to

$$\tilde{P} = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{d1} \\ p_{12} & p_{22} & \cdots & p_{d2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1d} & p_{2d} & \cdots & p_{dd} \end{pmatrix} \otimes \Xi \quad \text{where} \quad \Xi = \xi \otimes \xi \quad \text{and} \quad \xi = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_p \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Setting $d = 2$ and $p = 1$ leads to

$$\tilde{P} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \alpha_1^2$$

and

$$|\tilde{P} - \lambda I_2| = \begin{vmatrix} p_{11}\alpha_1^2 - \lambda & p_{21}\alpha_1^2 \\ p_{12}\alpha_1^2 & p_{22}\alpha_1^2 - \lambda \end{vmatrix} = \lambda^2 - (p_{11} + p_{22})\alpha_1^2\lambda + (p_{11}p_{22} - p_{12}p_{21})\alpha_1^4 \quad (5)$$

Setting (5) to zero gives $\lambda = \alpha_1^2$. Hence $\rho(\tilde{P}) < 1$ leads to $\alpha_1^2 < 1$ and thus $|\alpha_1| < 1$.

Hence our model features second order stationarity for

$$\begin{aligned} |\psi + \phi\mathcal{G}| &< 1 \\ \Leftrightarrow \psi &\in (-\phi\mathcal{G} - 1; -\phi\mathcal{G} + 1) \end{aligned}$$

For $\psi = 1$: $\phi\mathcal{G} \in (-2; 0)$. As $\mathcal{G} \in (0, 1)$, choosing $\phi = -1$ always yields a stationary model. \square

Remark $|\psi + \phi\mathcal{G}| < 1$ induces strict stationarity in our model as $\sum_{i=1}^d \pi_i \log |a_1(i)| = \log |a_1| = \log |\alpha_1| < 0$ for $|\psi + \phi\mathcal{G}| < 1$.

Appendix B: Tests of [Distaso \(2008\)](#) for dependent coefficients

The following table shows that the ALM^{ω^2} test by [Distaso \(2008\)](#) also works for dependent coefficients. For this, we model the random coefficient as an $AR(1)$ process. As the correlation is rather strong in our MS-STAR model, we show the results for high values of the autoregressive coefficient. We also choose different values of ω^2 , where $\omega^2 = 0$ leads to the size and otherwise the power of the test. For $\alpha = 0$ we have independent random coefficients, otherwise they are dependent. It can be seen that the test holds its nominal level of 5% for every value of α . The test is thus correctly sized also for dependent random coefficients. The power of the test even increases with stronger dependence of the random coefficient. Obviously, the power is not monotone, as the coefficients approach a random walk with $\alpha \rightarrow 1$. The results hold for moderate sample sizes such as $T = 200$ and $T = 500$ and are therefore applicable to our situation. We therefore believe that we can well apply the test of [Distaso \(2008\)](#) for our model selection procedure.

TABLE 1
Size and power of the ALM^{ω^2} test

ω^2	α				
	0	0.5	0.9	0.95	0.99
$T = 200$					
0	0.0500	0.0490	0.0521	0.0503	0.0494
0.00001	0.0512	0.0506	0.0494	0.0516	0.1583
0.0001	0.0443	0.0421	0.1044	0.2699	0.5270
0.001	0.0551	0.1256	0.6878	0.7879	0.7508
0.01	0.5943	0.7992	0.9777	0.9706	0.9220
$T = 500$					
0	0.0500	0.0501	0.0463	0.0482	0.0495
0.00001	0.0463	0.0476	0.0543	0.1365	0.5293
0.0001	0.0473	0.0616	0.5103	0.7589	0.8484
0.001	0.4202	0.6256	0.9767	0.9903	0.9355
0.01	0.9776	0.9977	1.0000	0.9996	0.9985

Notes: Results based on 10,000 replications. The simulated model is $q_t = \rho_t q_{t-1} + \varepsilon_t$, $\rho_t = 1 + \tilde{\rho}_t$, $\tilde{\rho}_t = \alpha \tilde{\rho}_{t-1} + \nu_t$, $\nu \sim N(0, \omega^2)$, $\tilde{\rho}_0 = 0$.

Appendix C: Subsample results prior to 2008

TABLE 2

Subsample results for the modified LM-tests on model specification

<i>Country</i>	$ALM\omega^2$	<i>(i)</i>		$ALM\rho$	<i>(ii)</i>		<i>Model</i>	<i>T</i>
		$\hat{\rho}$	$\hat{\sigma}_\varepsilon^2$		$\hat{\sigma}_\varepsilon^2$	$\hat{\omega}^2$		
Brazil	0.11	0.98949	0.00166	-1.19	0.00161	0.00075	1	337
Canada	3.91	1.00151	0.00017				3	420
Denmark	0.97	0.98649	0.00069	-1.56	0.00065	0.00187	2	420
Finland	1.34	0.98140	0.00063	-1.59	0.00055	0.00442	2	312
France	1.68	0.98306	0.00072				3	312
Germany	1.09	0.98427	0.00080	-1.52	0.00075	0.00228	1	312
Greece	0.01	0.98572	0.00076	-1.43	0.00076	0.00000	1	336
Italy	4.66	0.98342	0.00065				3	312
Japan	0.91	0.98674	0.00082	-1.51	0.00076	0.00223	1	420
Mexico	12.34	0.96606	0.00212				3	420
Netherlands	1.06	0.98203	0.00078	-1.73	0.00073	0.00214	2	312
Norway	1.40	0.98158	0.00059	-1.75	0.00053	0.00378	2	420
Portugal	0.53	0.98844	0.00086	-1.21	0.00078	0.00234	1	312
Spain	1.05	0.98788	0.00067	-1.33	0.00061	0.00219	1	312
Sweden	1.94	0.98877	0.00064				3	420
Turkey	18.55	0.99241	0.00169				3	420
UK	8.20	0.97694	0.00063				3	420

Notes: Data prior to 2008. States the test statistics (7) and (8) and underlying parameter estimates. Hypothesis (i): Based on critical values of Table 1 in [Distaso \(2008\)](#). Hypothesis (ii): Based on critical values of Table 3 in [Distaso \(2008\)](#). Bold values indicate that the Null cannot be rejected for $\alpha = 0.1$.

TABLE 3

Subsample estimation results and half-lives of the Markov-STAR model

Country	Parameter estimates							Half-lives			St. dev.
	$\hat{\mu}_1$	$\hat{\mu}_2$	\hat{p}_1	\hat{p}_2	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*	$\gamma = \gamma^*$	$\gamma = 0.01$	$\gamma = 1$	
Brazil	0.0013	-0.0022	0.8769	0.9801	0.0616	0.0088	0.03	7.8693	13.5138	1.2910	0.2683
Canada	0.0110	-0.0098	0.9428	0.9308	0.0070	0.0079	0.41	4.0183	22.1890	2.7554	0.1234
Denmark	0.0200	-0.0238	0.9449	0.9424	0.0147	0.0143	0.13	3.3479	10.2308	1.4608	0.1592
Finland	0.0204	-0.0203	0.9340	0.9452	0.0169	0.0128	0.64	1.9371	11.3410	1.6289	0.1437
France	0.0193	-0.0248	0.9404	0.9481	0.0176	0.0135	0.22	2.6575	10.1368	1.4329	0.1500
Germany	0.0224	-0.0237	0.9422	0.9301	0.0165	0.0168	0.14	3.1318	10.2436	1.4468	0.1639
Greece	0.0308	-0.0151	0.9360	0.9447	0.0172	0.0162	0.24	2.6618	8.9168	1.6124	0.1524
Italy	0.0273	-0.0149	0.9497	0.9528	0.0164	0.0152	0.43	2.2951	9.8079	1.6704	0.1459
Japan	0.0152	-0.0338	0.9536	0.9205	0.0162	0.0197	0.36	1.7547	6.2213	1.1931	0.2080
Mexico	0.0252	-0.0059	0.8831	0.9874	0.1010	0.0092	0.03	3.6493	5.1515	1.2168	0.1780
Netherlands	0.0204	-0.0262	0.9435	0.9516	0.0172	0.0140	0.17	2.7835	9.7585	1.3643	0.1573
Norway	0.0176	-0.0226	0.9400	0.9363	0.0142	0.0134	0.28	2.5817	11.2281	1.5838	0.1269
Portugal	0.0187	-0.0278	0.9197	0.9381	0.0207	0.0143	0.06	4.7878	11.0140	1.4193	0.1975
Spain	0.0272	-0.0162	0.9320	0.9504	0.0174	0.0146	0.28	2.4838	8.9882	1.5588	0.1846
Sweden	0.0238	-0.0177	0.9362	0.9469	0.0170	0.0132	0.30	2.5986	10.3146	1.5838	0.1961
Turkey	0.0299	-0.0065	0.8498	0.9750	0.0884	0.0188	0.00	41.6667	4.6022	0.9439	0.2529
UK	0.0190	-0.0207	0.9281	0.9299	0.0159	0.0149	0.44	2.3269	12.7994	1.6904	0.1250

Notes: Data prior to 2008. Estimation of model (6): $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_{s_t}$ with $\phi = 1$ and $\psi = -1$. Half-lives in years for a one standard deviation shock, where the first refers to the estimated value of γ , the second with an imposed γ of 0.01, and the third with an imposed γ of 1. Empirical standard deviations are given in the last column.

TABLE 4

Subsample logit regression results

Country	Constant	p-value	Outp. gap	p-value	Inflation	p-value	Uncert.	p-value	McF R^2	T
Brazil	-1.1269	0.0061	-1.4289	0.3175	-0.0643	0.5407	0.0120	0.0030	0.0620	325
Canada	-0.6188	0.0741	9.4221	0.0042	4.7021	0.0070	0.0058	0.0982	0.0541	408
Denmark	1.3047	0.0004	2.8823	0.0899	0.8644	0.5965	-0.0109	0.0034	0.0574	408
Finland	0.6584	0.1744	1.7049	0.3462	2.3812	0.1411	-0.0053	0.2752	0.0521	300
France	2.5609	0.0000	-1.6805	0.6349	2.5847	0.1380	-0.0215	0.0001	0.0861	300
Germany	1.0753	0.0358	5.2044	0.0257	-2.4437	0.2824	-0.0105	0.0485	0.0688	300
Greece	0.3606	0.4899	-4.3870	0.0214	2.3063	0.1688	-0.0101	0.0709	0.0535	324
Italy	-0.0553	0.9142	-1.5365	0.4844	0.1010	0.9449	-0.0067	0.2027	0.0397	300
Japan	1.9348	0.0000	2.0872	0.0229	-4.9357	0.0181	-0.0114	0.0017	0.0747	408
Mexico	-2.4107	0.0000	3.4719	0.1352	-1.1808	0.0237	0.0089	0.0284	0.1722	336
Netherlands	2.1194	0.0000	8.7446	0.0051	-1.4899	0.4797	-0.0174	0.0010	0.0970	300
Norway	1.1962	0.0009	0.2090	0.8119	1.1168	0.5011	-0.0096	0.0077	0.0454	408
Portugal	1.9498	0.0003	0.5018	0.6689	-0.6029	0.6694	-0.0145	0.0097	0.0704	300
Spain	-0.3781	0.4618	4.2706	0.1664	-0.1344	0.9333	-0.0012	0.8183	0.0354	300
Sweden	-0.0228	0.9476	2.9735	0.0905	0.6258	0.7034	-0.0026	0.4511	0.0352	408
Turkey	-2.1257	0.0017	7.0471	0.0004	0.3647	0.7244	0.0026	0.6080	0.3899	276
UK	0.4770	0.1690	1.4636	0.4466	2.9753	0.0239	-0.0036	0.2899	0.0407	408

Notes: Data prior to 2008. Based on (12) for the smoothed probabilities of model (6). Significant coefficients for $\alpha = 0.1$ in bold.

TABLE 5

Subsample estimation results of the Markov-STAR model with μ fixed

Country	$\hat{\mu}$	\hat{p}_1	\hat{p}_2	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*
Brazil	-0.0021	0.9801	0.8771	0.0088	0.0617	0.03
Canada	-0.0005	0.9841	0.8938	0.0034	0.0158	0.00
Denmark	0.0004	0.9626	0.8884	0.0102	0.0330	0.03
Finland	-0.0031	0.9908	0.8968	0.0087	0.0329	0.29
France	0.0016	0.9748	0.8776	0.0105	0.0361	0.00
Germany	-0.0005	0.9787	0.8805	0.0126	0.0392	0.00
Greece	-0.0028	0.9845	0.8917	0.0053	0.0339	0.09
Italy	-0.0043	0.9868	0.9115	0.0055	0.0322	0.38
Japan	0.0030	0.9990	0.8983	0.0045	0.0343	0.24
Mexico	-0.0058	0.9870	0.8818	0.0092	0.1057	0.02
Netherlands	-0.0011	0.9724	0.8806	0.0129	0.0381	0.03
Norway	0.0000	0.9719	0.8947	0.0085	0.0308	0.12
Portugal	0.0013	0.9821	0.8760	0.0099	0.0398	0.07
Spain	-0.0039	0.9713	0.8754	0.0099	0.0367	0.00
Sweden	-0.0001	0.9769	0.8860	0.0076	0.0323	0.00
Turkey	-0.0052	0.9733	0.8485	0.0184	0.0926	0.00
UK	0.0004	0.9698	0.8759	0.0097	0.0344	0.36

Notes: Data prior to 2008. Estimation of model (6), where now $q_t = \mu_t + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_s$, with $\phi = 1$ and $\psi = -1$.

TABLE 6

Subsample logit regression results for μ fixed

Country	Constant	p-value	Outp. gap	p-value	Inflation	p-value	Uncert.	p-value	McF R ²	T
Brazil	1.1213	0.0062	1.2708	0.3730	0.0687	0.5128	-0.0118	0.0034	0.0610	325
Canada	-1.2724	0.0034	-4.5195	0.2535	-3.3220	0.0850	-0.0023	0.6064	0.0663	408
Denmark	0.2069	0.6456	5.5103	0.0067	4.1680	0.0380	-0.0129	0.0073	0.0686	408
Finland	0.0914	0.8686	6.6496	0.0009	-5.0442	0.0032	-0.0111	0.0519	0.0933	300
France	1.1636	0.0383	6.8358	0.0715	0.7173	0.6885	-0.0186	0.0018	0.0770	300
Germany	1.1251	0.0314	2.8371	0.2359	5.6993	0.0158	-0.0130	0.0169	0.0869	300
Greece	-1.2220	0.0641	6.7250	0.0003	-3.4002	0.0549	-0.0084	0.2454	0.1094	324
Italy	0.9385	0.1661	16.2789	0.0000	0.7940	0.6451	-0.0208	0.0045	0.1895	300
Japan	-1.3159	0.0327	0.5087	0.7242	-2.1673	0.3215	-0.0090	0.1693	0.0654	408
Mexico	2.2959	0.0000	-2.7796	0.2281	1.1417	0.0290	-0.0077	0.0591	0.1661	336
Netherlands	0.9124	0.0606	-1.8501	0.5296	2.0511	0.3153	-0.0114	0.0235	0.0479	300
Norway	0.5358	0.2199	0.8752	0.3516	1.3325	0.4745	-0.0150	0.0014	0.0552	408
Portugal	0.5277	0.3151	0.3416	0.7662	2.3182	0.0918	-0.0077	0.1675	0.0447	300
Spain	1.1262	0.0449	6.4103	0.0428	-5.8893	0.0004	-0.0194	0.0009	0.1126	300
Sweden	0.1528	0.7240	2.1322	0.2882	2.9163	0.1299	-0.0119	0.0104	0.0412	408
Turkey	2.1936	0.0007	-6.7048	0.0006	-0.9818	0.3308	-0.0063	0.2087	0.4016	276
UK	-0.3523	0.3120	3.0275	0.1247	4.7870	0.0013	0.0014	0.6855	0.0470	408

Notes: Data prior to 2008. Based on (12) for the smoothed probabilities of model (6), where μ_t is fixed. Significant coefficients for $\alpha = 0.1$ in bold.

TABLE 7

Subsample estimation results of the Markov-STAR model with σ fixed

Country	$\hat{\mu}$	$\hat{\mu}_2$	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\sigma}$	γ^*	LR
Brazil	0.1008	-0.0079	0.9416	0.9899	0.0304	0.11	0.27
Canada	0.0108	-0.0102	0.9360	0.9384	0.0075	0.45	0.01
Denmark	0.0203	-0.0235	0.9460	0.9410	0.0145	0.14	0.01
Finland	0.0216	-0.0191	0.9388	0.9288	0.0150	0.61	0.04
France	0.0217	-0.0222	0.9401	0.9245	0.0158	0.20	0.02
Germany	0.0220	-0.0240	0.9431	0.9317	0.0167	0.13	0.00
Greece	0.0311	-0.0149	0.9401	0.9443	0.0166	0.25	0.01
Italy	0.0279	-0.0145	0.9529	0.9531	0.0157	0.43	0.03
Japan	0.0146	-0.0351	0.9530	0.9318	0.0175	0.36	0.03
Mexico	0.3603	-0.0027	0.9990	0.9990	0.0295	0.24	0.50
Netherlands	0.0207	-0.0258	0.9504	0.9366	0.0158	0.11	0.01
Norway	0.0186	-0.0215	0.9408	0.9331	0.0138	0.36	0.01
Portugal	0.0327	-0.0153	0.9348	0.9428	0.0191	0.18	0.02
Spain	0.0294	-0.0150	0.9385	0.9529	0.0157	0.29	0.04
Sweden	0.0263	-0.0159	0.9445	0.9455	0.0151	0.31	0.04
Turkey	0.1649	-0.0056	0.9990	0.9964	0.0302	0.01	0.15
UK	0.0194	-0.0203	0.9310	0.9286	0.0154	0.42	0.01

Notes: Data prior to 2008. Estimation of model (6), where now $q_t = \mu_{s_t} + \phi q_{t-1} + \psi \mathcal{G}(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with $\phi = 1$ and $\psi = -1$.

TABLE 8

Subsample mean squared errors for 1-period-ahead forecasts

Country	Markov-STAR (fixed regimes)	Markov-STAR (averaged)	Random walk (without)	Random walk (with drift)
Brazil	0.8239	0.8244	0.4234	0.4219
Canada	0.0607	0.0570	0.0455	0.0454
Denmark	0.2246	0.1821	0.1242	0.1238
Finland	0.1140	0.1022	0.0869	0.0874
France	0.1146	0.0956	0.0636	0.0634
Germany	0.1306	0.0992	0.0667	0.0664
Greece	0.1408	0.1170	0.0839	0.0836
Italy	0.1155	0.1032	0.0781	0.0779
Japan	0.3076	0.2829	0.1626	0.1632
Mexico	0.2872	0.2705	0.2317	0.2310
Netherlands	0.1290	0.1107	0.0718	0.0716
Norway	0.2067	0.1795	0.1288	0.1283
Portugal	0.1273	0.1042	0.0701	0.0696
Spain	0.1306	0.1137	0.0790	0.0793
Sweden	0.2863	0.2644	0.1534	0.1528
Turkey	0.3879	0.3779	0.3531	0.3518
UK	0.2245	0.1675	0.1126	0.1120

Notes: Data prior to 2008, forecasting period starts in 1990. For the model $\hat{q}_{t+1} = \hat{\mu}_t + \phi q_t + \psi \mathcal{G}(q_t; \hat{\gamma}, c) q_t$ with $\phi = 1$ and $\psi = -1$, where $\hat{\mu}_t$ is based on a fixed regime if $\hat{\xi}_{i,t+1|T} \geq .5$, or a $\hat{\xi}_{i,t+1|T}$ -weighted average. Compared to the naive approach $\hat{q}_{t+1} = q_t$ and a random walk with drift $\hat{q}_{t+1} = \alpha + q_t$.

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